Structural Vibration – Sheet 2

1e)

This example is to shows how where you linearize a system can affect the resulting EOM. In the original hand written solutions the equations are linearized first and then manipulated. In this solution the equations are manipulated first and then linearized (slightly more difficult). Both answers are “correct”, where multiplying the solution given here by [1 1;0 L] will give you the same solution found in the handwritten solutions.

Applying F=ma to the two masses (see exercise sheet solution) gives:

$$T\sin\theta - kx = m_1\ddot{x} \qquad (1)$$

$$-T\sin\theta = m_2(\ddot{x} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta) \qquad (2)$$

$$T\cos\theta - m_2 g = m_2(L\ddot{\theta}\sin\theta + L\dot{\theta}^2\cos\theta) \qquad (3)$$

Adding (1) and (2) and re-arranging gives:

$$(m_1 + m_2)\ddot{x} + m_2(L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta) + kx = 0$$

Adding (2)x$\cos\theta$ and (3)x$\sin\theta$, and re-arranging gives:

$$m_2 L\ddot{\theta} + m_2\ddot{x}\cos\theta + m_2 g\sin\theta = 0$$

For small θ, $\sin\theta = \theta, \cos\theta = 1$ and neglecting products of small quantities (i.e. $\dot{\theta}^2\theta = \dot{\theta}^2 = 0$) the last two equations can be approximated to:

$$(m_1 + m_2)\ddot{x} + m_2 L\ddot{\theta} + kx = 0$$

$$m_2\ddot{x} + m_2 L\ddot{\theta} + m_2 g\theta = 0$$

These equations of motion can be written in matrix form as follows:

$$\begin{pmatrix} m_1 + m_2 & m_2 L \\ m_2 L & m_2 L^2 \end{pmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & m_2 gL \end{pmatrix}\begin{pmatrix} x \\ \theta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$